Conference

Foundations in Mathematics: Modern Views

Booklet of Abstracts

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Organizing committee

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Sponsors

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Contents

THORSTEN ALTENKIRCH

UNIVERSITY OF NOTTINGHAM

Introduction to (Homotopy) Type Theory

Thursday, 3.30 pm – 4.45 pm

Homotopy Type Theory is a new foundation of Mathematics which generalizes Martin-Löf Type Theory to higher dimensions, that is non-trivial equality types. It introduces some new basic principles, such as the univalence axiom, which identifies isomorphic types. It also enables us to conduct an abstract version of homotopy theory which has exciting but largely unexplored applications to dependently typed programming. The talk builds on the HoTT book (available for free: `https://homotopytypetheory.org/book/`) but will also cover some more recent developments like cubical type theory.

MARIANNA ANTONUTTI MARFORI

LMU MUNICH (MCMP)

De re and de dicto knowledge in mathematics: two case studies from mathematical practice

Friday, 2 pm – 3.15 pm

This talk will introduce the scope distinction used in other areas of philosophy between *de re* and *de dicto* propositional attitudes in order to illuminate debates in mathematical practice. It is often the case that certain mathematical proofs are praised as being more elegant, explanatory, deep, etc., than other proofs of the same theorem. It is then natural to ask what this 'additional epistemic value' consists of. I will examine two such case studies from abstract algebra and discrete mathematics, and I will argue that the 'additional epistemic value' of the proofs in question can be seen as '*de re* content' of those proofs. In order to highlight the 'de re content' of different proofs of those theorems I will use results from reverse mathematics. The philosophical import of reverse mathematics is often understood in ontological terms, as showing us the existential commitments that attend our acceptance of certain mathematical theorems. In this talk, however, I will use these results in the service of mathematical epistemology.

Roughly, a proof provides *de dicto* knowledge of a mathematical statement if it provides knowledge of a purely existential statement, whereas a proof provides *de re* knowledge when it carries additional information concerning the identity of the objects that are proven to exist. Consider the following two theorems, (T1) that every countable commutative ring has a maximal ideal, and (T2) that every countable commutative ring has a prime ideal. The former is equivalent to $\mathsf{ACA0}$, while the latter is equivalent to $\mathsf{WKL0}$, with both equivalences being provable in $\mathsf{RCA0}$. The former is proved by giving a definition of a set that is a maximal ideal, and proving that set to exist using arithmetical comprehension. From this proof, we gain *de re* knowledge of (T1) because the proof gives an explicit construction of a maximal ideal. However, we could have either *de re* or *de dicto* knowledge of (T2) depending on the proof we use to establish the conclusion. Using the proof for (T1) we gain *de re* knowledge of (T2), since the proof gives an explicit construction of a maximal ideal, and every maximal ideal is a prime ideal. Using Weak König's Lemma, instead, such an explicit construction is not possible: the proof merely establishes the existence of a prime ideal, and thus only provides *de dicto* knowledge of (T2).

I will then discuss a second case study concerning two different proofs of the theorem that every bipartite graph has a König cover (for any bipartite graph G, there exists a matching F in G and a set C consisting of one vertex from each edge in F such that C is a cover of G), where the merely existential proof reverses to $\mathsf{ATR0}$ and the proof providing an explicit definition of the graph in question reverses to Π^1_1-$\mathsf{CA0}$, and I will argue that a similar analysis can be carried out as in the first case study.

I will suggest that this way of articulating the *de re*/*de dicto* distinction lines up with certain model theoretic properties of subsystems of second order arithmetic, such as the existence of certain kinds of minimal models. Furthermore, I will argue that there are good reasons not to identify the *de re* content of a proof with its constructive content, nor with its predicative content. Finally, I will argue that this way of characterising the 'additional epistemic value' of certain proofs fits with value judgments made in ordinary mathematical practice.

STEPHAN HARTMANN

LMU MUNICH (MCMP)

What is Mathematical Philosophy?

Thursday, 9.30 am – 10.45 am

Mathematical Philosophy is the study of philosophical problems with the help of mathematical methods. In this talk, I will give three examples that illustrate that power of mathematical methods in philosophy. By doing so, I will also illustrate that mathematical philosophy is relevant beyond pure philosophy as it often addresses problems at the interface between philosophy, science and public policy.

Luca Incurvati

University of Amsterdam (ILLC)

Iteration and dependence again

Thursday, 11 am – 12.15 pm

In the first part of the talk I clarify what is at stake in the debate between accounts of the iterative conception based on the notion of metaphysical dependence and the minimalist account I put forward in earlier work. Special attention will be paid to the relationship between this debate and the debate between actualist and potentialist accounts of the cumulative hierarchy. In the second part of the talk, I use the distinctions drawn in the first part of the talk to assess an objection recently leveled by Mark Gasser against structuralist accounts of mathematics.

Yurii Khomskii

University of Hamburg

Regularity and Definability in the Real Number Continuum

Thursday, 2 pm – 3.15 pm

In the study of the real number continuum, regularity properties of sets of real numbers play a central role, having many applications in various areas of mathematics. For instance, a subset of the continuum is "Lebesgue-measurable" if one can determine its "size" or "volume" in a coherent way.

Using the Axiom of Choice one can construct sets that do not satisfy a given regularity property (e.g., are not Lebesgue-measurable). However, one cannot produce an explicit examples of such a set. What exactly does that mean?

In descriptive set theory, sets of real numbers are classified in a complexity hierarchy, according to the complexity of the formula in second-order number theory that defines the given set. Sets low according to this hierarchy (Borel and analytic sets) satisfy all the standard regularity properties, but if we look at more complex sets, we obtain statements which are independent of ZFC (the standard axiomatisation of set theory).

In this talk I will present this interplay between regularity and definability of sets of real numbers, mentioning some classical results as well as some results related to my own work.

Paolo Mancosu

UC Berkeley

Neologicist foundations: inconsistent abstraction principles and part whole

Saturday, 11 am – 12.15 pm

[Joint work with Benjamin Siskind, UC Berkeley]

Neologicism emerges in the contemporary debate in philosophy of mathematics with Wright's book *Frege's Conception of Numbers as Objects* (1983). Wright's project was to show the viability of a philosophy of mathematics that could preserve the key tenets of Frege's approach, namely the idea that arithmetical knowledge is analytic. The key result was the detailed reconstruction of how to derive, within second order logic, the basic axioms of second order arithmetic from Hume's principle (HP) (and definitions). This has led to a detailed scrutiny of so-called abstraction principles, of which Basic Law V (BLV) and HP are the two most famous instances. As is well known, Russell proved that BLV is inconsistent. BLV has been the only example of an abstraction principle from (monadic) concepts to objects giving rise to inconsistency, thereby making it appear as a sort of monster in an otherwise regular universe of abstraction principles free from this pathology. We show that BLV is part of a family of inconsistent abstractions. The main result is a theorem to the effect that second-order logic formally refutes the existence of any function F that sends concepts into objects and satisfies a "part-whole" relation. In addition, we study other properties of abstraction principles that lead to formal refutability in second order logic.

Iosif Petrakis

LMU Munich

Bishop's constructivism in foundations and practice of mathematics

Saturday, 9.30 am – 10.45 am

Half a century ago Errett Bishop published his seminal book "Foundations of constructive analysis", a reconstruction of large parts of (informal) mathematical analysis within intuitionistic logic. Bishop's informal system BISH, which underlies his constructivism, is neutral and considerably powerful. It is neutral, as it lies in the common territory of intuitionistic, recursive, and classical mathematics. It is is considerably powerful, as large areas of classical mathematics have a constructive counterpart within it.

Although Bishop wanted to revolutionize with his simple foundations the standard mathematical practice, his work influenced dramatically only the foundations of mathematics. Martin-Loef's type theory, constructive set theory, and Feferman's explicit mathematics are some of the formal systems developed in the 70's and the 80's in order to formalize BISH.

In this talk we review the role of Bishop's constructivism in modern foundations of mathematics and present some significant features of its current practice.

Lavinia Picollo

LMU Munich (MCMP)

Frege's Logicism and the Neologicists

Friday, 3.30 pm – 4.45 pm

To prove the possibility of *a priori* knowledge of numbers, Frege attempted to reduce arithmetic to logic. This project is nowadays known as "logicism". More precisely, Frege showed that the axioms of second-order arithmetic can be derived in a second-order system known as "Frege Logic" (FL). We introduce FL and sketch Frege's derivation. Unfortunately, as we show turning to Russell's paradox, this system is inconsistent, i.e. trivial.

Attempts to repair FL and to rescue the logicist project were abandoned due to Gödel's Incompleteness Theorems and the rise of set theory. However, towards the end of the XX century the core ideas of logicism were revived by the so-called neologicists. It has been noticed that the axioms of second-order arithmetic follow from a single theorem of FL, i.e. Hume's Principle (HP), which turns out to be consistent. This result is known as "Frege's Theorem". Thus, if HP were *a priori* knowable, as has been argued by the neologicists, the possibility of *a priori* knowledge of numbers could be finally established. We present and discuss the main present neologicists arguments in favour of the *a priori* knowability of HP and issues the project faces.

HELMUT SCHWICHTENBERG

LMU MUNICH

Proof and computation

Friday, 11 am – 12.15 pm

We discuss a logical framework TCF (theory of computable functionals) suitable for the extraction of computational content from proofs. TCF can be seen as a variant of HA^{ω} (Heyting's arithmetic in all finite types). The main differences are (i) TCF has the Scott-Ershov partial continuous functionals as its intended model; (ii) the term part of TCF is an extension T^{+} of Gödel's system T with functions defined by possibly non-terminating rules; (iii) (co)inductive predicates with their least (and greatest) fixed point axioms are allowed.

Following Kolmogorov we then view formulas as problems asking for a solution, called "realizers" by Kleene and Kreisel. For this to make sense we first have to define what a realizer for a (co)inductive predicate I applied to some arguments $\vec{r}$ is. The natural choice here is to take a witness of our knowledge that I holds for $\vec{r}$. This is an object in the free algebra build from constructors corresponding to I's defining clauses; for coinductive predicates this object can be infinite (a "stream"). We then can define what the "type" of a formula is, i.e., the type of a solution to the problem posed by the formula. The soundness theorem states that from a proof M of a formula A of type ρ we can extract a term $\mathrm{et}(M)$ in T^{+} of the same type ρ such that (provably in TCF) the term $\mathrm{et}(M)$ is a realizer of A.

As an example we consider a constructive proof of the fan theorem for "coconvex" bars, based on recent work of Josef Berger and Gregor Svindland. Using an appropriate concept of uniform coconvexity and viewing paths as streams, we extract from the formalized proof a simple algorithm computing the upper bound. Other examples deal with real number computation based on "Gray-code", a binary number system where neighboring values differ in one digit only.

WOLFGANG WOHOFSKY

UNIVERSITY OF KIEL

Generalized Cantor Space and Compactness

Friday, 9.30 am – 10.45 am

I will talk about the Generalized Cantor Space 2^{κ}, for κ being a regular (uncountable) cardinal (with κ equals ω being the classical Cantor space). I will discuss topological properties such as compactness (which hold true in the case of ω) and give an example from my own research: an argument to prove a certain classical theorem seems to essentially use the "compactness of 2^{ω}" at first sight, but by trying to generalize the question to 2^{κ}, it turns out that actually less (namely the "inaccessibility of ω") is sufficient for the proof of the theorem.